1. Write your name at the top of each paper that you hand in.
2. There are 3 problems. Partial credit may be given only if your work is clearly shown.
3. Start each problem on a new page of paper.
4. This will be due in class at 9:30 AM, Tuesday, Apr. 12.
5. All that you hand in must be your own work (Honor Code).

1 [30 points] Special Relativity - Minimum Energy

What is the minimum proton energy, in the lab frame of reference, needed to produce anti-protons, $\overline{p}$, by the reaction $p+p \rightarrow p+p+(p+\overline{p})$? Assume that initially, in the lab frame of reference, that one proton is moving and the other is at rest. Answer in terms of the rest mass of the proton, m_p and the speed of light in a vacuum, c.

2 [35 points] Linear Fixed Point Analysis

The dynamics of a particular system can be modeled by the following set of ordinary differential equations (ODEs)

$$\dot{x} = y \tag{2.1}$$

$$\dot{y} = -\sin x - y. \tag{2.2}$$

2.1 (6) Fixed Points

Find the fixed points x_0, y_0, for this system. In other words, find the values of x and y that cause $\dot{x} = 0$ and $\dot{y} = 0$. How many fixed points are there?

2.2 (10) Linearize the Equations of Motion Near the Fixed Points

By defining $\mu = x - x_0$, and $\nu = y - y_0$, write the linearized (expanded) equations of motion, that are valid near the fixed points, x_0, y_0. Your answer should be of the form $\begin{pmatrix} \dot{\mu} \\ \dot{\nu} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \mu \\ \nu \end{pmatrix}$, where A, B, C, and D are numbers. By using an appropriate notation you can write the linearized equations of motion for all the fixed points in one set of equations of this form.

2.3 (10) Find the Eigen Values and Eigen Vectors

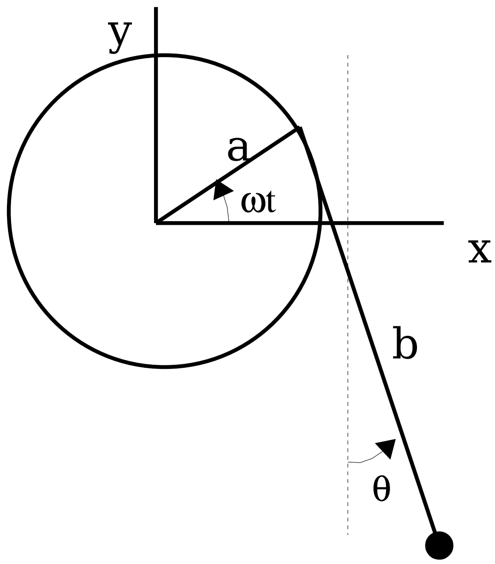
By substituting a solution of the form $\begin{pmatrix} \mu \\ \nu \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{\lambda t}$ into your linearized equations of motion, find the general solution of the linearized equations of motion near the fixed points. Substitute sin and cos functions where appropriate to remove any complex numbers.

2.4 (9) Draw Phase Plots

Draw the y vs x phase plot of the flow of this system near the fixed points.

3 [35 points] Hamiltonian

Seeing that canonical transformations and quantum mechanics are beyond the scope of this class, in this problem we will show, again, how the Hamiltonian method of mechanics is not as useful as Lagrangian methods. You can check your answer indirectly with Thornton page 243 Equation 7.35. There is a copy of Thornton in the SPS Meeting Room, Room 222, Robeson Hall.



A simple pendulum is driven by having its pivot point connected to the edge of a rotating disk that rotates at an angular frequency ω. The length of the pendulum is b. The radius of the rotating disk is a. The point mass on the end of the pendulum has a mass m.

3.1 (9) Lagrangian

In terms of m, a, b, ω, θ, $\dot{\theta}$, and t, write an expression for the Lagrangian for this system.

3.2 (8) Canonical Conjugate Momentum

Find the canonical momentum that is conjugate (paired) to θ. That is, find $p_\theta \equiv \frac{\partial L}{\partial \dot{\theta}}$, as a function of m, a, b, ω, θ, $\dot{\theta}$, and t.

3.3 (9) Hamiltonian

In terms of m, a, b, ω, θ, p_θ, and t, write the Hamiltonian, $H(\theta, p_\theta, t)$ for this system.

3.4 (9) Hamilton's Equations of Motion

Find the equations of motion for θ and p_θ.