Fixed Points 7 HW7

1.

$$\dot{x} = y^3 - 8 \qquad \begin{matrix}\dot{x}=0\\ \dot{y}=0\end{matrix} \Rightarrow \begin{matrix} y_0^3 - 8 = 0 \\ x_0 + 2y_0 = 0\end{matrix} \Rightarrow \boxed{\begin{matrix} y_0 = 2 \\ x_0 = -4 \end{matrix}} \qquad \boxed{\begin{matrix} x_0 = -4 \\ y_0 = 2 \end{matrix}}$$

$$\dot{y} = x + 2y$$

2.

$$\dot{x} = f(x,y) \qquad \dot{x} \simeq \left.\frac{\partial f}{\partial x}\right|_{\substack{x=x_0\\y=y_0}} (x-x_0) + \left.\frac{\partial f}{\partial y}\right|_{\substack{x=x_0\\y=y_0}} (y-y_0)$$

$$\dot{y} = g(x,y)$$

$$\dot{y} \simeq \left.\frac{\partial g}{\partial x}\right|_{\substack{x=x_0\\y=y_0}} (x-x_0) + \left.\frac{\partial g}{\partial y}\right|_{\substack{x=x_0\\y=y_0}} (y-y_0)$$

$$\Rightarrow \begin{matrix} \dot{x} \simeq 0 + 3y_0^2 (y-y_0) = 12(y-y_0) \\ \dot{y} \simeq (x-x_0) + 2(y-y_0) = (x-x_0) + 2(y-y_0) \end{matrix}$$

$$u = x - x_0 \qquad v = y - y_0 \qquad \Rightarrow \boxed{\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & 12 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}}$$

3.

$$\vec{q} \equiv \begin{pmatrix} u \\ v \end{pmatrix} \qquad \vec{q} = \vec{a} e^{\lambda t} \qquad \dot{\vec{q}} = \lambda \vec{a} e^{\lambda t}$$

$$\text{plug} \Rightarrow \lambda \vec{a} = \begin{pmatrix} 0 & 12 \\ 1 & 2 \end{pmatrix} \vec{a} \Rightarrow \begin{pmatrix} -\lambda & 12 \\ 1 & 2-\lambda \end{pmatrix} \vec{a} = 0$$

$$\Rightarrow \begin{vmatrix} -\lambda & 12 \\ 1 & 2-\lambda \end{vmatrix} = 0 \Rightarrow -\lambda(2-\lambda) - 12 = 0$$

$$\Rightarrow \lambda^2 - 2\lambda - 12 = 0 \Rightarrow \lambda = \frac{2 \pm \sqrt{4 + 4(12)}}{2}$$

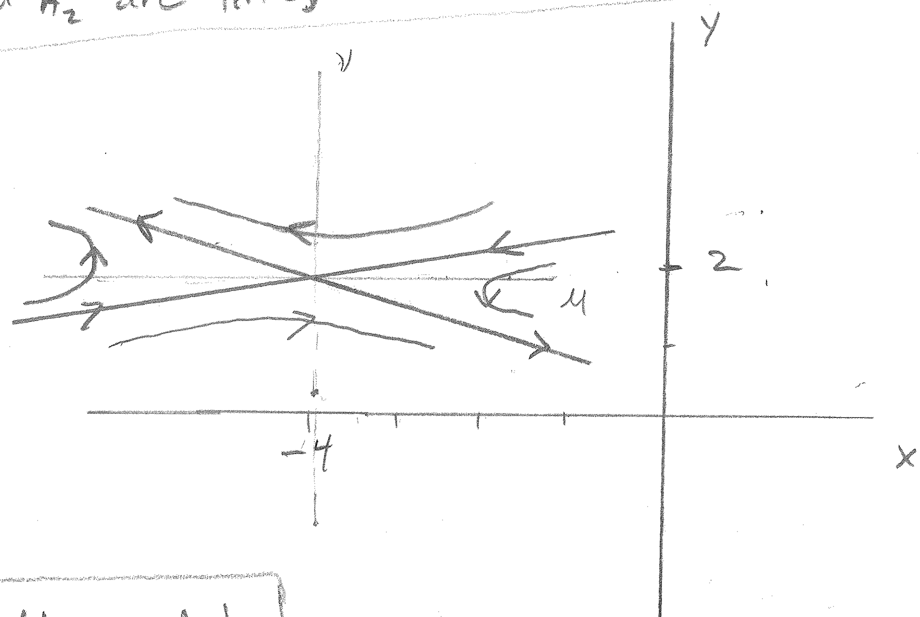
$$\Rightarrow \lambda = 1 \pm \sqrt{13} \qquad (1 \pm \sqrt{13}) a_1 + 12 a_2 = 0$$

$$\Rightarrow \vec{a} = \begin{pmatrix} 12 \\ -1 \mp \sqrt{13} \end{pmatrix}$$

$$\begin{pmatrix} x+4 \\ y-2 \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix} = A_1 \begin{pmatrix} 12 \\ -(1+\sqrt{13}) \end{pmatrix} e^{(\sqrt{13}+1)t} + A_2 \begin{pmatrix} 12 \\ \sqrt{13}-1 \end{pmatrix} e^{-(\sqrt{13}-1)t}$$

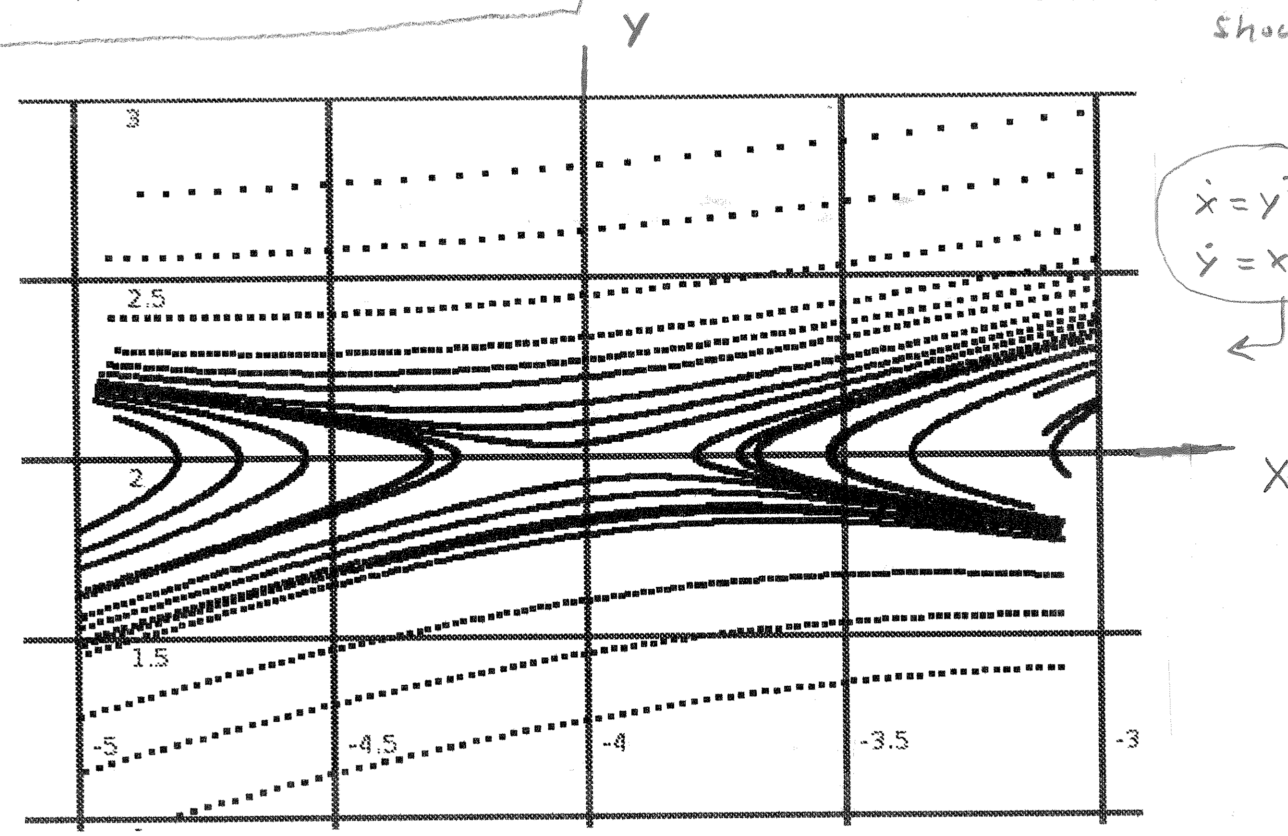
A_1 and A_2 are integration constants

4.



It's a saddle point.
It's unstable.

From numical simulation
Flow Directions are not shown.



$\dot{x} = y^3 - 8$

$\dot{y} = x + 2y$