The dynamics of a particular system can be modeled by the following set of ordinary differential equations (ODEs)

$$\dot{x} = y^3 - 8 \quad (0.1)$$
$$\dot{y} = x + 2y. \quad (0.2)$$

You will see there is only one fixed point for this system.

1 Fixed Point

Find the fixed point x_0, y_0, for this system. In other words, find the values of x and y that cause $\dot{x} = 0$ and $\dot{y} = 0$.

2 Linearize the Equations of Motion Near the Fixed Point

By defining $\mu = x - x_0$, and $\nu = y - y_0$, write the linearized (expanded) equations of motion, that are valid near the fixed point, x_0, y_0. Your answer should be of the form $\begin{pmatrix} \dot{\mu} \\ \dot{\nu} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \mu \\ \nu \end{pmatrix}$, where A, B, C, and D are numbers.

3 Find the Eigen Values and Eigen Vectors

By substituting a solution of the form $\begin{pmatrix} \mu \\ \nu \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{\lambda t}$ into your linearized equations of motion, find the general solution of the linearized equations of motion near the fixed point. Your answer should be of the form $\begin{pmatrix} \mu \\ \nu \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} e^{\lambda_1 t} + \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} e^{\lambda_2 t}$, where λ_1 and λ_2 are the two eigen values and $\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}$ and $\begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}$ are the two eigen vectors.

4 Draw a Phase Plot

Draw the y vs x phase plot of the flow of this system near the fixed point. Would you call this fixed point a stable or unstable fixed point?