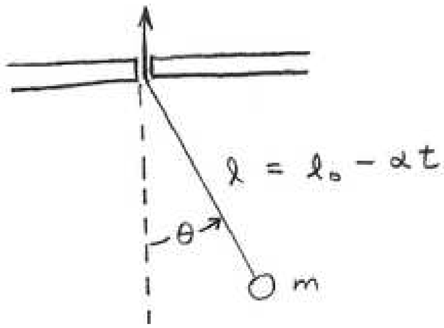

A simple plane pendulum consists of a mass m attached to a string of length l. The suspension point of the pendulum remains fixed, as the length, l, is shortened at a constant rate α, as shown above. The pendulum has a length l_0 at time $t = 0$. In this problem we'll find the equations of motion for the pendulum using the Lagrangian and the Hamiltonian method, and we'll look at the energy in the system.

1 Lagrangian

In terms of θ, $\dot{\theta}$, m, α, l_0, and t, write an expression the Lagrangian for this system.

2 Lagrange's Equations of Motion

Using your Lagrangian, find Lagrange's equations of motion.

3 Canonical Conjugate Momentum

Find the canonical momentum that is conjugate (paired) to the generalize coordinate θ. That is find $p_\theta \equiv \frac{\partial L}{\partial \dot{\theta}}$, as a function of θ, $\dot{\theta}$, m, α, l_0, and t.

4 Hamiltonian

In terms of θ, p_θ, m, α, l_0, and t, write the Hamiltonian, $H(\theta, p_\theta, t)$, for this system. There should be no $\dot{\theta}$ in the Hamiltonian for this system. Your dynamical variables are now the canonical pair θ and p_θ.

5 Hamilton's Equations of Motion

Find the equations of motion for θ and p_θ and show that these equations of motion, for θ and p_θ, are equivalant to the equations of motion from section 2.

6 Energy vs Hamiltonian

In terms of m and α, find the difference between the total energy and the Hamiltonian.