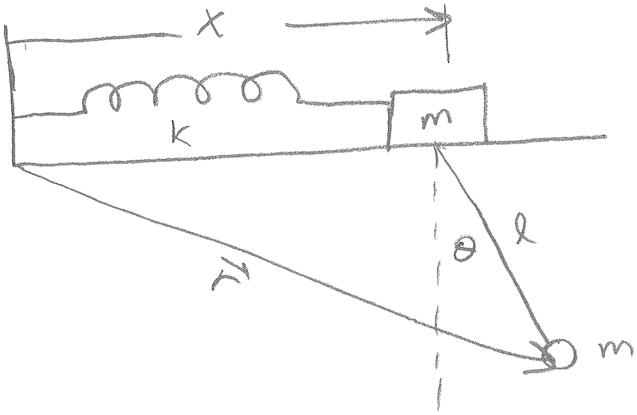
HW 3

Problem Additional



$$\frac{k}{m} = \frac{g}{l} \Rightarrow k = \frac{mg}{l}$$

define $\omega_0^2 = \frac{k}{m}$

$$T = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m\dot{r}^2 = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m\dot{x}^2 + m l \dot{x}\dot{\theta} + \frac{1}{2} m l^2 \dot{\theta}^2$$

$$= m\dot{x}^2 + m l \dot{x}\dot{\theta} + \frac{1}{2} m l^2 \dot{\theta}^2$$

$$\Rightarrow T = \frac{1}{2} (\dot{x} \quad l\dot{\theta}) \begin{pmatrix} 2m & m \\ m & m \end{pmatrix} \begin{pmatrix} \dot{x} \\ l\dot{\theta} \end{pmatrix} \equiv \frac{1}{2} \tilde{\dot{\vec{q}}} \overleftrightarrow{M} \dot{\vec{q}}$$

$$V = \frac{1}{2} k x^2 + mgl(1-\cos\theta) \simeq \frac{1}{2} k x^2 + \frac{1}{2} k l^2 \theta^2$$

$$\Rightarrow V = \frac{1}{2} (x \quad l\theta) \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} x \\ l\theta \end{pmatrix} \equiv \frac{1}{2} \tilde{\vec{q}} \overleftrightarrow{K} \vec{q}$$

equation of motion are

$$\overleftrightarrow{M} \ddot{\vec{q}} + \overleftrightarrow{K} \vec{q} = 0$$

Assume $\vec{q} = \vec{a} \cos(\omega t - \delta)$

$\therefore \ddot{\vec{q}} = -\omega^2 \vec{a} \cos(\omega t - \delta)$

$= -\omega^2 \vec{q}$ $\qquad \vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

$$\Rightarrow \left(-\omega^2 \overleftrightarrow{M} + \overleftrightarrow{K}\right) \vec{a} = 0 \quad ①$$

divide by m $\Rightarrow$

$$(-\omega^2 2 + \omega_0^2) a_1 + (-\omega^2) a_2 = 0 \quad ②$$

$$(-\omega^2) a_1 + (-\omega^2 + \omega_0^2) a_2 = 0$$

$$② \Rightarrow \frac{a_1}{a_2} = \frac{-\omega^2 + \omega_0^2}{\omega^2} \qquad ① \Rightarrow \frac{a_1}{a_2} = \frac{\omega^2}{\omega_0^2 - 2\omega^2}$$

$$\det\left| -\frac{\omega^2}{m}\overleftrightarrow{M} + \frac{\overleftrightarrow{k}}{m} \right| = 0$$

$$\Rightarrow \left(-2\omega^2 + \omega_0^2\right)\left(-\omega^2 + \omega_0^2\right) - \left(\omega^2\right)^2 = 0$$

$$\Rightarrow 2\left(\omega^2\right)^2 - \left(\omega^2\right)^2 - \omega^2\left(2\omega_0^2 + \omega_0^2\right) + \left(\omega_0^2\right)^2 = 0$$

$$\Rightarrow \left(\omega^2\right)^2 - 3\omega^2\omega_0^2 + \left(\omega_0^2\right)^2 = 0$$

$$\Rightarrow \omega^2 = \frac{3\omega_0^2}{2} \pm \frac{1}{2}\sqrt{9(\omega_0^2)^2 - 4(\omega_0^2)^2} \Rightarrow \omega^2 = \frac{3 \pm \sqrt{5}}{2}\omega_0^2$$

$$① \Rightarrow \frac{q_1}{q_2} = \frac{\frac{3 \pm \sqrt{5}}{2}\omega_0^2}{\omega_0^2 - (3 \pm \sqrt{5})\omega_0^2} = \frac{\frac{1}{2}(3 \pm \sqrt{5})}{1 - 3 \mp \sqrt{5}} = \frac{1}{2}\frac{(3 \pm \sqrt{5})}{-2 \mp \sqrt{5}}\frac{(-2 \pm \sqrt{5})}{(-2 \pm \sqrt{5})}$$

$$= \frac{-6 \mp 2\sqrt{5} \pm 3\sqrt{5} + 5}{2(4 - 5)} = \frac{-1 \pm \sqrt{5}}{-2} = \frac{1 \mp \sqrt{5}}{2}$$

Mode

$$\vec{Q}_1 = \begin{pmatrix} \frac{1 - \sqrt{5}}{2} \\ 1 \end{pmatrix} \cos\left(\sqrt{\frac{3 + \sqrt{5}}{2}}\,\omega_0 t - \delta_1\right)$$

$$\vec{Q}_2 = \begin{pmatrix} \frac{1 + \sqrt{5}}{2} \\ 1 \end{pmatrix} \cos\left(\sqrt{\frac{3 - \sqrt{5}}{2}}\,\omega_0 t - \delta_2\right)$$

Not normalized