Relativity 8 HW8

1) $$\frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_2^2} + \frac{\partial^2 \psi}{\partial x_3^2} - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = 0 \quad (1)$$

change variables from $\psi(x_1, x_2, x_3, t)$ to $\psi'(x_1', x_2', x_3', t')$.

$\psi(x_1, x_2, x_3, t) = \psi'(x_1', x_2', x_3', t')$

$$\frac{\partial^2 \psi}{\partial x_1^2} = \gamma^2 \frac{\partial^2 \psi'}{\partial x_1'^2} - \gamma^2 \frac{v}{c^2}\frac{\partial^2 \psi'}{\partial x_1' \partial t'} - \gamma^2 \frac{v}{c^2}\frac{\partial^2 \psi'}{\partial t' \partial x_1'} + \left(\frac{\gamma v}{c^2}\right)^2 \frac{\partial^2 \psi'}{\partial t'^2}$$

$$\frac{\partial \psi}{\partial x_2} = \frac{\partial x_2'}{\partial x_2}\frac{\partial \psi'}{\partial x_2'} = \frac{\partial \psi'}{\partial x_2'} \Rightarrow \frac{\partial^2 \psi}{\partial x_2^2} = \frac{\partial^2 \psi'}{\partial x_2'^2}$$

$$\frac{\partial^2 \psi}{\partial x_3^2} = \frac{\partial^2 \psi'}{\partial x_3'^2}$$

$$\frac{\partial \psi}{\partial t} = \frac{\partial x_1'}{\partial t}\frac{\partial \psi'}{\partial x_1'} + \overset{0}{\cancel{\frac{\partial x_2'}{\partial t}}}\frac{\partial \psi'}{\partial x_2'} + \overset{0}{\cancel{\frac{\partial x_3'}{\partial t}}}\frac{\partial \psi'}{\partial x_3'} + \frac{\partial t'}{\partial t}\frac{\partial \psi'}{\partial t'}$$

$$= (-\gamma v)\frac{\partial \psi'}{\partial x_1'} + \gamma \frac{\partial \psi'}{\partial t'}$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial t^2} = \left[(-\gamma v)\frac{\partial}{\partial x_1'} + \gamma \frac{\partial}{\partial t'}\right]\left[(-\gamma v)\frac{\partial \psi'}{\partial x_1'} + \gamma \frac{\partial \psi'}{\partial t'}\right]$$

$$= \gamma^2 v^2 \frac{\partial^2 \psi'}{\partial x_1'^2} - \gamma^2 v \frac{\partial^2 \psi'}{\partial x_1' \partial t'} - \gamma^2 v \frac{\partial^2 \psi'}{\partial t' \partial x_1'} + \gamma^2 \frac{\partial^2 \psi'}{\partial t'^2}$$

writing (1) in terms of primed system

$$\Rightarrow \gamma^2 \frac{\partial^2 \psi'}{\partial x_1'^2} - \cancel{\gamma^2 \frac{v}{c^2}\frac{\partial^2 \psi'}{\partial x_1' \partial t'}} - \cancel{\gamma^2 \frac{v}{c^2}\frac{\partial^2 \psi'}{\partial t' \partial x_1'}} + \left(\frac{\gamma v}{c^2}\right)^2 \frac{\partial^2 \psi'}{\partial t'^2}$$

$$+ \frac{\partial^2 \psi'}{\partial x_2'^2} + \frac{\partial^2 \psi'}{\partial x_3'^2} - \frac{1}{c^2}\left[\gamma^2 v^2 \frac{\partial^2 \psi'}{\partial x_1'^2} - \cancel{\gamma^2 v \frac{\partial^2 \psi'}{\partial x_1' \partial t'}} - \cancel{\gamma^2 v \frac{\partial^2 \psi'}{\partial t' \partial x_1'}} + \gamma^2 \frac{\partial^2 \psi'}{\partial t'^2}\right]$$

$$= 0$$

$$\Rightarrow \underbrace{\gamma^2\left(1-\frac{v^2}{c^2}\right)}_{1}\frac{\partial^2\psi'}{\partial x_1'^2} + \frac{\partial^2\psi'}{\partial x_2'^2} + \frac{\partial^2\psi'}{\partial x_3'^2} - \frac{1}{c^2}\underbrace{\left[\gamma^2\left(1-\frac{v^2}{c^2}\right)\right]}_{1}\frac{\partial^2\psi'}{\partial t'^2} = 0$$

$$\Rightarrow \frac{\partial^2\psi'}{\partial x_1'^2} + \frac{\partial^2\psi'}{\partial x_2'^2} + \frac{\partial^2\psi'}{\partial x_3'^2} - \frac{1}{c^2}\frac{\partial^2\psi'}{\partial t'^2} = 0 \qquad \text{QED}$$

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b) using $x_1' = x_1 - vt \quad x_2' = x_2 \quad x_3' = x_3 \quad t' = t$

$$\Rightarrow \frac{\partial\psi}{\partial x_1} = \frac{\partial x_1'}{\partial x_1}\frac{\partial\psi'}{\partial x_1'} + 0 + 0 + 0$$

$$\Rightarrow \frac{\partial^2\psi}{\partial x_1^2} = \frac{\partial^2\psi'}{\partial x_1'^2}, \quad \frac{\partial^2\psi}{\partial x_2^2} = \frac{\partial^2\psi'}{\partial x_2'^2}, \quad \frac{\partial^2\psi}{\partial x_3} = \frac{\partial^2\psi}{\partial x_3'^2}$$

$$\frac{\partial\psi}{\partial t} = \frac{\partial x_1'}{\partial t}\frac{\partial\psi'}{\partial x_1'} + 0 + 0 + \frac{\partial t'}{\partial t}\frac{\partial\psi'}{\partial t'} = -v\frac{\partial\psi'}{\partial x_1'} + \frac{\partial\psi'}{\partial t'}$$

$$\frac{\partial^2\psi}{\partial t^2} = \left(-v\frac{\partial}{\partial x_1'} + \frac{\partial}{\partial t'}\right)\left(-v\frac{\partial\psi'}{\partial x_1'} + \frac{\partial\psi'}{\partial t'}\right)$$

$$= v^2\frac{\partial^2\psi'}{\partial x_1'^2} - v\frac{\partial^2\psi'}{\partial t'\partial x_1'} - v\frac{\partial^2\psi'}{\partial x_1'\partial t'} + \frac{\partial^2\psi'}{\partial t'^2}$$

$$(1) \Rightarrow \frac{\partial^2\psi'}{\partial x_1'^2} + \frac{\partial^2\psi'}{\partial x_2'^2} + \frac{\partial^2\psi'}{\partial x_3'^2} - \frac{1}{c^2}\left[v^2\frac{\partial^2\psi'}{\partial x_1'^2} - 2v\frac{\partial^2\psi'}{\partial t'\partial x_1'} + \frac{\partial^2\psi'}{\partial t'^2}\right] = 0$$

$$\Rightarrow \frac{\partial^2\psi'}{\partial x_1'^2} + \frac{\partial^2\psi'}{\partial x_2'^2} + \frac{\partial^2\psi'}{\partial x_3'^2} - \frac{1}{c^2}\frac{\partial^2\psi'}{\partial t'^2} + \frac{2v}{c^2}\frac{\partial^2\psi'}{\partial t'\partial x_1'} - \frac{v^2}{c^2}\frac{\partial^2\psi'}{\partial x_1'^2} = 0$$

Not in the same form $\therefore$ not invariant